Minimal supersymmetric SO(10) GUT with doublet Higgs

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We propose a simple supersymmetric SO(10) GUT model with only doublet Higgs scalars. The fermion mass problem is naturally solved by a new one-loop diagram. R-parity is conserved implying a stable LSP which can explain the dark matter of the universe. There are two contributions to the neutrino masses, one of which depends on the quark mass squared while the other is independent and similar to the type II see-saw mechanism. In the latter case $b - \tau$ unification implies large neutrino mixing angles. The baryon asymmetry of the universe is explained through leptogenesis.

The supersymmetric grand unified theories (GUTs) turned out to be the most natural extension of the standard model, in which all the gauge interactions unify to a single interaction. The simplest supersymmetric GUT with the gauge group $SU(5)$ suffers from several problems like proton decay, unification of coupling constants, fermion masses and mixings. There are solutions to these problems, but each of these solutions makes the theory more complex. The most natural choice for the simplest GUT then turns out to be the one based on the group $SO(10)$. All fermions, including the right-handed neutrinos belong to the 16-dimensional spinor representation of the group. At higher energies the theory predicts parity invariance and the generators of the group include left-right symmetry and $B - L$ symmetry [1], which makes the theory even more attractive. As a consequence of the spontaneous breaking of the $B - L$ symmetry, the neutrino mass comes out to be small naturally via the see-saw mechanism [2].

There are many versions of the supersymmetric $SO(10)$ GUT with varying predictions. The most popular version of the theory includes triplet Higgs scalars to break the left-right symmetry and simultaneously give Majorana masses to neutrinos. These triplets belong to a $\mathbf{126}$-dimensional representation and a $\overline{\mathbf{126}}$-plet representation is required for anomaly cancellation. Recently a minimal $SO(10)$ GUT has been proposed [3, 4, 5, 6], in which the $\mathbf{126}$ explains light neutrino masses and mixing [4], solves the problem of wrong prediction for fermion masses in the $SO(10)$ GUT [3], namely, $m_\mu = m_s$ and $m_e = m_d$, and conserves R-parity at low energies [5] implying a stable LSP which then solves the dark matter problem.

In the present article we propose yet another simpler supersymmetric $SO(10)$ GUT with only doublet Higgs scalars. Models with doublet Higgs scalars have been discussed in the past [7, 8], but here we present a minimal model with doublet Higgs scalar, which has many advantages. The $\mathbf{126}$ and $\overline{\mathbf{126}}$ of Higgs scalars are now replaced by $\mathbf{16}$ and $\overline{\mathbf{16}}$ of Higgs scalars and one $SO(10)$ singlet fermion per generation. This model has three more coupling constants, but has 217 less superfields compared to the minimal models with triplet Higgs scalars [6]. In addition, this model can be motivated by string theory, in which case the coupling constants will be determined by the string tension. This model can also be embedded in orbifold $SO(10)$ GUTs naturally [8]. Leptogenesis is also possible in this scenario, which is difficult in the minimal SUSY $SO(10)$ models with triplet Higgs scalars.

In the present model there is a new natural solution to the fermion mass problem. Neutrino masses come out naturally light in the observed range. Nutrino mixing angles can also be very large. In one case the $b - \tau$ unification ensures this large mixing angle and is consistent with small quark mixing angles. R-parity is also conserved in this model at low energies so that the lightest superparticle is stable, which can then solve the dark matter problem of the universe.

Except for the contribution of $\mathbf{126}$, which contains the triplet Higgs scalars, there are many similarities between the present model and the minimal $SO(10)$ GUT, which has been studied extensively during the past few years. In the present model fermions of each generation (including a right-handed neutrino) belong to the 16-plet spinor representation, which transforms under the Pati-Salam subgroup ($G_{422} \equiv SU(4)_c \times SU(2)_L \times SU(2)_R$) as,

$$\Psi_i \equiv \mathbf{16} = (\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}).$$

$i = 1, 2, 3$ is the generation index. We also include one heavy $SO(10)$ singlet fermion superfield $S_a \equiv \mathbf{1} = (\mathbf{1}, \mathbf{1}, \mathbf{1})$ per generation ($a = 1, 2, 3$). When we use four numbers (x, x, x, x), it would represent the subgroup $G_{3221} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.

The Higgs superfields in this model are,

$$
\begin{aligned}
\Phi \equiv \mathbf{210} &= (\mathbf{1}, \mathbf{1}, \mathbf{1}) + (\mathbf{6}, \mathbf{2}, \mathbf{2}) + (\mathbf{15}, \mathbf{3}, \mathbf{1}) + (\mathbf{15}, \mathbf{1}, \mathbf{3}) \\
&\quad + (\mathbf{15}, \mathbf{1}, \mathbf{1}) + (\mathbf{10}, \mathbf{2}, \mathbf{2}) + (\overline{\mathbf{10}}, \mathbf{2}, \mathbf{2}) \\
\Gamma \equiv \mathbf{16} &= (\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \\
\overline{\Gamma} \equiv \overline{\mathbf{16}} &= (\overline{\mathbf{4}}, \mathbf{2}, \mathbf{1}) + (\mathbf{4}, \mathbf{1}, \mathbf{2}) \\
H \equiv \mathbf{10} &= (\mathbf{6}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{2}, \mathbf{2}).
\end{aligned}
$$

The Higgs sector in our model is the smallest compared to all the existing models of supersymmetric $SO(10)$ GUT.

Before we write down the superpotential we need to discuss the question of R-parity and matter parity in this model. Since the fermions contained in the superfield Ψ and also the left-right symmetry breaking scalar superfield Γ transform as $\mathbf{16}$-plet superfields, we need to distinguish them using matter parity. Although R-parity

and matter parity for ordinary fermions and scalars are well defined, to include the heavier particles we need to generalize the definition as

$$M = (-1)^{3(B-L)+\chi}, \qquad (1)$$

where χ is a new quantum number and $\chi = 1$ for the fields S_a, Γ and $\overline{\Gamma}$. Similarly, we should also extend the definition of the R-parity and define it as,

$$R = (-1)^{3(B-L)+2S+\chi}. \qquad (2)$$

With this extension, the R-parity now becomes consistent with matter parity, since the fermion superfields S_a have $B-L = 0$. We then need to impose that all interactions should be invariant under M-parity. This discrete symmetry is not broken at any stage.

We can then write down the superpotential with the scalar fields Φ, Γ, $\overline{\Gamma}$ and H, which is invariant under M-parity as,

$$\begin{aligned} W = & \frac{m_\Phi}{4!} \Phi^2 + m_\Gamma \Gamma \overline{\Gamma} + m_H H^2 + \frac{\lambda}{4!} \Phi^3 \\ & + \frac{\eta}{4!} \Phi \Gamma \overline{\Gamma} + H (\alpha \Gamma \Gamma + \overline{\alpha} \overline{\Gamma} \overline{\Gamma}). \end{aligned} \qquad (3)$$

The number of parameters in this potential is same as in the minimal SO(10) GUT with triplet Higgs scalars [6].

The $SO(10)$ GUT symmetry is broken to the left-right symmetric group G_{3221} by the vev of the $\Phi_1 = (1, 1, 1)$ and $\Phi_{15} = (15, 1, 1)$ components of Φ at the GUT scale M_U. The left-right symmetry is broken by the doublet Higgs scalar

$$\xi_R \equiv (1, 1, 2, -1) \subset (4, 1, 2) \subset \overline{\Gamma},$$

when the neutral component of ξ_R acquires a vev $u_R = \langle \xi_R^0 \rangle$ at some intermediate scale. This carries the same quantum number as the right-handed neutrinos. The vanishing of the D-term is ensured by giving equal vevs to u_R and $\overline{u_R} = \langle \overline{\xi_R} \rangle$. Since this field carry $B-L = -1$, this cannot give Majorana masses to the neutrinos, but that is not a problem as we shall discuss later. The left-handed counterpart of this field ($\xi_L \equiv (1, 2, 1, -1) \subset (4, 2, 1) \subset \Gamma$) acquires a induced vev after the electroweak symmetry breaking, $u_L = \langle \xi_L^0 \rangle \sim 100$ GeV due to the interaction $\Gamma\Gamma H$. During the left-right symmetry breaking at a scale u_R, another component of Φ acquires an induced vev $\Phi_3 = (15, 1, 3)$ due to the couplings $\Phi \Gamma \overline{\Gamma}$.

The super potential is very similar to that with triplet Higgs and can be minimized to get the required solution in a similar way [6]. For the symmetry breaking near the GUT scale, we shall not include the electroweak symmetry breaking Higgs scalar H. In terms of the vevs the superpotential is then given by

$$\begin{aligned} W = & m_\Phi(\Phi_1^2 + 3\Phi_{15}^2 + 6\Phi_3^2) + 2\lambda(\Phi_{15}^3 + 3\Phi_1\Phi_3^2 \\ & + 6\Phi_{15}\Phi_3^2) + m_\Gamma \overline{u_R} u_R + \eta \overline{u_R} u_R (\Phi_1 + 3\Phi_{15} + 6\Phi_3). \end{aligned}$$

This equation has been solved and the possible solutions allow the symmetry breaking scenario discussed

in the previous para. For a choice of parameters [6], $3 \lambda m_\Gamma \sim -2 \eta m_\Phi$, it allows the $SO(10)$ symmetry to break down to the group G_{3221} at the scale $M_U \sim m_\Phi$ and the symmetry breaking G_{3221} to the standard model at a scale $u_R \sim m_\Gamma$.

All the Higgs scalars are even under matter parity and all fermions are odd. As a result matter parity is not broken by the vev of any scalars. The R-parity for the scalar components of the scalar superfields are always even. So, none of the vevs break the R-parity. Although the fields Γ and $\overline{\Gamma}$ carry $B-L = -1$, because of the χ quantum number the R-parity is not broken by the vevs of these fields. As a result at low energy it is not possible to generate any R-parity violating interactions. Thus the lightest superparticle will remain stable and this can solve the problem of dark matter of the universe.

We shall now discuss the question of fermion masses. The superpotential containing the Yukawa couplings is

$$W = h_{ij}\Psi_i\Psi_j H + y_{ia}\Psi_i S_a \overline{\Gamma} + M_{ab} S_a S_b + H.c. \qquad (4)$$

All fields are chiral superfields and hence there are no terms with H^*. We imposed M-parity to write down this superpotential. It is possible to diagonalise both y_{ij} and M_{ab} simultaneously. Then y_{ij} contains 3 parameters, but h_{ij} contains 6 real parameters. Compared to the triplet Higgs models, the mass terms for the singlets S_a are the only additional parameters in the present model. Since we diagonalised M_{ab}, so that $M_{ab} = M_a \delta_{ab}$ (we shall assume a hierarchy $M_1 < M_2 < M_3$ and denote the scale by $M_S = M_3$), this introduces three additional parameters compared to the minimal SO(10) model with triplet Higgs scalars. However, for models with triplet Higgs, type II see-saw [9] contribution can work only if the model is extended to include another **54** Higgs, which then introduces six more parameters [10]. In the present model no extra field is required to get the good features of the type II see-saw mechanism and also to generate a lepton asymmetry of the universe.

The two neutral components of $H \equiv (1, 2, 2, 0)$ acquires vevs $\kappa_{u,d} \sim 10^2$ GeV, which gives masses to the up and down sectors respectively. $\kappa_u \equiv (1, -1/2, 1/2, 0)$ gives masses to the up-quarks and Dirac masses to the neutrinos while $\kappa_d \equiv (1, 1/2, -1/2, 0)$ gives masses to the down-quarks and charged leptons (here we have given the $SU(2)$ quantum numbers). Since this Higgs does not distinguish between quarks and leptons, the quark-lepton symmetry of the $SO(10)$ group will give a wrong mass relation $m_\mu = m_s$ and $m_e = m_d$. In the $SO(10)$ GUTs with triplet Higgs scalars, the $(15, 2, 2)$ component of **126** receives an induced vev, which can solve this fermion mass problem. In the present scenario there are no such representations which can solve this problem. Fortunately, there is a one loop diagram which can solve this problem, which we discuss below.

Although there are no **126** representation which can contribute to the fermion masses, there is an effective term $\frac{y_{ia}y_{aj}}{M_a} \Psi_i \Psi_j \overline{\Gamma} \overline{\Gamma}$ which comes after integrating out the S_a field. The $\Gamma \Gamma$ combination behaves like a **126**
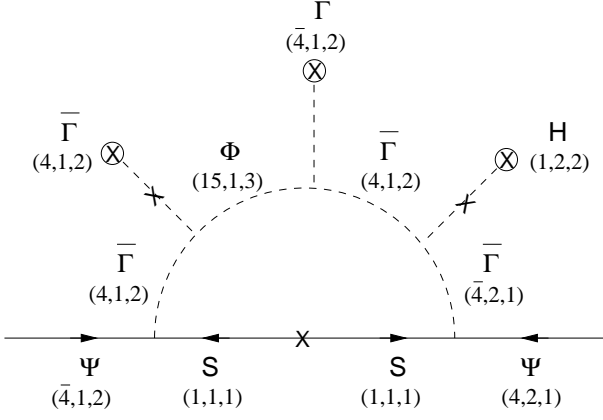
FIG. 1: One loop diagram contributing to the fermion masses which breaks the quark-lepton symmetry. The components of the fields are given below the names of the fields.

field, and also contains a combination $(15, 2, 2)$, but only one component of this effective field gets *vev* which contributes to neutrino masses only. So, this cannot solve the problem. Fortunately there is another effective term, which can solve this problem,

$$\frac{\zeta}{M_X^2} \, \Psi_i \, \Psi_j \, \Gamma \, \overline{\Gamma} \, H, \qquad (5)$$

where ζ is some effective coupling constant and M_X is one of the heavy scales. In figure 1 we presented the one loop diagram that can contribute to the fermion masses from this effective term. The quantum numbers of the component fields are also presented to demonstrate that the $(15, 1, 3)$ term enters the diagram, and hence this contributes only as effective $(15, 2, 2)$ to the fermion masses.

The contribution to the fermion masses from the one loop diagram of figure 1 comes out to be,

$$\tilde{M}_{u,d} = \beta \, y_{ia} \, y_{aj} \, D \, \frac{u_R^2}{M_X^2} \, \kappa_{u,d} = D \, y_{ia} \, y_{aj} \, m_{u,d}, \quad (6)$$

where $D = \text{diag}[1, 1, 1, -3]$ is a diagonal matrix acting on the $SU(4)_c$ space, which is the $SU(3)_c$ singlet of **15**. In this estimate, the mass of the heavier field between S and Φ_3 comes in the denominator. Although most of the components of the field Φ have mass of the order of the GUT scale, the $(15, 1, 3)$ component (Φ_3) remains light. The neutral component of $(15, 1, 3)$ picks up an induced *vev* of order u_R (as discussed earlier). So, by survival hypothesis this component remains light and its mass will be about $m_{15} \sim (1 - 10) \, u_R$. So, the mass M_X in the denominator of this expression represents the heavier mass between M_S and m_3.

For neutrino masses an interesting solution will correspond to $M_S \sim u_R$, so we take $M_X \sim 10 \, u_R$. Then including the loop factors we get $m_{u,d} \sim 100$ MeV as required. The fermion masses are then given by,

$$M_u = h\kappa_u + y^2 m_u, \qquad M_d = h\kappa_d + y^2 m_d,$$

$$M_\nu^D = h\kappa_u - 3y^2 m_u, \qquad M_\ell = h\kappa_d - 3y^2 m_d,$$

This can then solve the fermion mass problem.

We shall now discuss the question of neutrino masses. Since there are no Higgs superfield with $B - L = 2$, there are no tree level Majorana neutrino masses. However, the S_a singlets are Majorana particles and they couple to the neutrinos, which can then give Majorana masses to the neutrinos. The neutrino mass matrix in the basis $[\nu_L \quad \nu_R \quad S]$ now becomes

$$M_\nu = \begin{pmatrix} 0 & h \, \kappa_u & y \, u_L \\ h \, \kappa_u & 0 & y \, u_R \\ y \, u_L & y \, u_R & M_a \end{pmatrix}. \qquad (7)$$

The entries h, y and M_a are 3×3 matrices (with M_a diagonal). The eigenvalues will now depend on the scales of M_S and u_R. In the limit $M_S > y \, u_R$, two of them will be large with eigenvalues M_S and $y^2 \, u_R^2/M_S$. The third one, which is essentially the left-handed neutrino, will be light and the mass is given by,

$$M_{\nu ij} = \frac{h^2 \kappa_u^2}{v_R} - C h y^2 \frac{\kappa_u^2}{v_R}. \qquad (8)$$

where $v_R = y^2 u_R^2/M_a$ and C is a constant, which determines which term dominates. The first term is similar to the conventional see-saw contribution of the $SO(10)$ GUTs with triplet Higgs and proportional to quark mass squared, while the second contribution is similar to the type II see-saw.

If the second term dominates, we can follow the logic [4] similar to that of the triplet Higgs models to show that $b - \tau$ unification will imply large neutrino mixing angles. From the expression of the mass matrices we observe $M_\nu \propto hy^2 \propto M_d^2 - M_\ell^2$ to a leading order. In the basis of diagonal charged leptons and up quarks the quark mixing is contained in M_d. Consider only second and third generation for explaining the logic with $m_s = m_\mu = 0$. If the small quark mixing is assumed to vanish, then $M_\nu \propto \begin{pmatrix} 0 & 0 \\ 0 & m_b^2 - m_\tau^2 \end{pmatrix}$. Thus with $b - \tau$ unification large mixing in the neutrino mass matrix will be possible as in the type II see-saw model of the triplet Higgs $SO(10)$ GUT.

In the limit of $M_S < y \, u_R$, there are two heavy neutrinos with masses $M_S \pm y \, u_R$ and are almost degenerate. The light neutrino will now have contribution dominantly from the second term. If we consider $y \, u_R \sim M_S \sim 10^{13}$ GeV and $h \, \kappa_u \sim y \, u_L \sim 100$ GeV, then the neutrino masses come out to be of the order of eV. Including the coupling constants it will then be possible to obtain the required neutrino masses with maximal mixings.

We now turn to the question of leptogenesis [11]. The main constraint for thermal leptogenesis in this scenario is the masses of the right handed neutrinos and the singlets S_a. When $M_S > y \, u_R$, the right-handed neutrinos can decay to left-handed neutrinos and Higgs bi-doublets through lepton number violating interaction due to the

effective Majorana mass. This can then generate a lepton asymmetry of the universe. However, since the masses of the right-handed neutrinos cannot be smaller than 10^{13} GeV in this scenario or the minimal $SO(10)$ GUT with triplet Higgs, the gravitino problem will not allow this mechanism to work.

In the present model this problem is solved when $M_S < y\, u_R$. Now the decays of both the right-handed neutrinos as well as the singlet fermions S_a will contribute to leptogenesis. The singlets S_a can decay into light leptons and Higgs doublets $(1, 2, 1, -1) \subset (4, 2, 1) \subset \Gamma$. The mixing of $(1, 2, 1, -1)$ with ordinary Higgs bi-doublets $(1, 2, 2, 0)$ after the left-right symmetry breaking will give rise to lepton number violation. CP violation comes from new diagrams. In this case the right-handed neutrinos and S_a will be almost degenerate. As a result resonant leptogenesis [12] will become possible. So, the leptogenesis can take place at a much lower temperature.

The right-handed neutrinos and the S_a will decay at a higher temperature around 10^{13} GeV. Then inflation will erase all asymmetry. However, after the reheating temperature 10^{10} GeV, the inflaton decay will produce particles as heavy as 10^{13} GeV but the number density of these particles will be less. So, after reheating right-handed neutrinos and S_a will be produced, whose decay can generate lepton asymmetry. Since the number density is less, there will be large suppression in the amount of asymmetry. On the other hand, since the masses of the right-handed neutrinos and the singlet fermions are almost degenerate, there will be resonance and large enhancement of the produced asymmetry. These two effects will now make leptogenesis possible and it will be possible to generate the required amount of baryon asymmetry of the universe before the electroweak phase transition.

In summary, we presented a minimal supersymmetric $SO(10)$ GUT with only doublet Higgs scalar. This model gives correct symmetry breaking pattern, solves the fermion mass problem, preserves R-parity so that the LSP can solve the dark matter problem, predicts light Majorana neutrinos with large mixing angle (in one case with $b - \tau$ unification) and able to generate a baryon asymmetry of the universe.

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